Knots in Algebra, Geometry and Physics
MATH 249 Selected Topics in Geometry and Topology

Clarisson Rizzie P. Canlubo
clarisson31415.wordpress.com

Abstract
Knots and related objects such as links, braids and ribbons are among the most interesting objects in mathematics for many reasons. Hidden in their simplicity lies a sophistication that endures great minds of mathematics. Mathematics and physics have never been more entangled than the past century and a great many of these entanglements revolve around knots. Meanwhile, even within mathematics many areas that seem to have no profound connections converge to knot theory. The main focus of this course, apart from putting knots and its related objects into spotlight, is to explore the many deep connections within mathematics, and between mathematics and physics through knots. We will try to keep the prerequisite of this course to a minimum, but since most of the connections require advanced topics in math the students are encouraged to do further readings.

1 Course Outline

I. Ouverture

1. Knots, links, ribbons, and braids
2. Operations on knots and its related objects
3. Categories, functors, and invariants
4. Numerical invariants; Gauss integral formula

II. The Jones Polynomial

5. Markov’s Braid Theorem
6. Braid groups and the Burau representations
7. Representations of the symmetric group
8. Hecke algebras
9. Ocneanu trace
10. The Jones polynomial
11. Other polynomial invariants

III. Quantum Invariants
12. Quantum groups
13. The Quantum Yang-Baxter Equation and R-matrices

IV. Vassiliev Invariants
14. Definitions and properties
15. Examples of Vassiliev invariants
16. Relations with knot polynomials

V. Kontsevich’s Universal Knot Invariant
17. Chord diagrams
18. Braided tensor categories and ribbon categories
19. The universal knot invariant

VI. Additional Topics
20. Knot complements and knot groups
21. Temperley-Lieb algebras
22. Topological Quantum Field Theories
23. Khovanov Cohomology

2 References

1If time permits we will tackle these topics.